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Navigation Solutions for the Repeated-Intercept Mission with Constrained Maneuver Time

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This paper provides navigation solutions to perform the repeated-intercept mission with a constrained time of maneuver. The repeated-intercept (flyby) mission can be performed by a small, highly maneuverable microsatellite designed to fly past a target satellite at high relative velocity. Because of non-Keplerian orbital perturbations, the microsatellite must perform a small maneuver at or shortly after the intercept to adjust its orbit to permit another intercept approximately one revolution later. During the course of that orbit the microsatellite uses midcourse guidance and terminal guidance to refine its trajectory and the intercept geometry. The mission repeats, allowing the microsatellite to image the target satellite from various angles and under multiple lighting conditions, providing an on-orbit inspection capability without matching orbital planes. This paper presents a strategy for selection of the magnitude and direction of the initial maneuver to provide the greatest number of flyby opportunities within constrained budgets of time and fuel.

Nomenclature

$\boldsymbol{e}$	=	vector of error components
f_i	=	function of the control variable used in the cost function
k	=	scalar factor used to scale control vector adjustments in the steepest-descent method
$L(\boldsymbol{u})$	=	scalar cost function
Q	=	weighting matrix giving accuracy of observations or weights of error components
$\boldsymbol{R}_{\text{ms}}(t_{\text{int}})$	=	position vector of the microsat
$R_{\text{ms},x}$	=	Cartesian components of the microsat position vector
$\boldsymbol{R}_{\text{tgt}}(t_{\text{int}})$	=	position vector of the target
$R_{\text{tgt},x}$	=	Cartesian components of the target position vector
$\boldsymbol{r}_f$	=	vector miss distance
s_i	=	scaling parameter in the cost function
T	=	matrix of sensitivities of error components with respect to control variables
t_f	=	final time
t_{int}	=	time of intercept
t_0	=	initial time
t_0^-	=	instant just before burn at the initial time
$\boldsymbol{u}$	=	vector of control variables
$V_{\text{ms},x}$	=	Cartesian component of the microsat velocity vector
$V_{\text{tgt},x}$	=	Cartesian component of the target velocity vector
$\boldsymbol{X}_{\text{ms}}$	=	state vector of the microsat
$\boldsymbol{X}_{\text{tgt}}$	=	state vector of the target
$\Delta \boldsymbol{V}$	=	delta-v maneuver
ΔV_x	=	Cartesian components of the delta-v maneuver
$\Phi(t_f, t_0)$	=	state transition matrix
Φ_{sub}^{u-r}	=	upper right 3×3 submatrix of the state transition matrix

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Introduction

CURRENTLY, the ability of microsatellites (microsats) to perform on-orbit inspection missions requires a coorbital rendezvous with a target satellite. Although this allows the persistent monitoring of a particular target satellite, the microsat must be launched directly into the plane of the target satellite's orbit. This effectively prevents a microsat from inspecting multiple targets.

Conversely, microsats also have the ability to perform an entirely different form of inspection mission not requiring a coplanar orbit: the repeated-intercept mission. The repeated-intercept mission requires the microsat to repeatedly fly past a target satellite, usually with a very high relative velocity. A flyby occurs nominally once per orbit, although other frequencies are possible and may be operationally useful. The repeated-intercept mission allows the microsat to image the target from multiple perspectives and in various lighting conditions.

The principle drawback of this inspection method is the high rate of relative velocity between the microsat and target. The primary advantage of this method is the wide range of target satellites that are within the range of a single microsat. A microsat can wait in a standby, or parking, orbit until commanded to image a target; it does not need to launch into the target's orbit plane. This also obviates the need for a rapid launch-on-demand capability. Because the microsat does not require matched velocities, it can potentially image targets in a very wide range of orbits. In fact, it can be shown that a single microsat in an appropriate parking orbit would be in a position to repeatedly intercept any target in low-Earth orbit (LEO) within 24 hours of command.

Problem Statement

A fundamental characteristic of the repeated-intercept mission is that each intercept occurs at nearly the same location in space. This characteristic causes other satellite navigation methods to fail when applied to the repeated-intercept problem. The specific cause of these failures will be discussed in a later section. This paper presents a means of determining navigation solutions to perform the repeated-intercept mission. The microsat initially performs a maneuver or series of maneuvers to affect the initial intercept. The navigation required for this initial intercept does not suffer from the difficulty described previously and has been well-studied [1–11]. This paper assumes that this initial intercept has already occurred and focuses on the navigation required to cause the intercept to repeat. With Keplerian dynamics, a simple period-matching maneuver would generate subsequent repeats. However, real-world non-Keplerian perturbations could generate an unacceptable miss distance after even a single orbit preventing the microsat from imaging the target at the required range. The microsat performs a small maneuver at or immediately after the initial intercept to adjust its orbit to permit another intercept approximately one revolution later. This maneuver includes both the required period-matching component as well as a component needed to account for the non-Keplerian perturbations. During the course of that orbit the microsat generally requires midcourse guidance and almost certainly requires terminal guidance. This paper does not address midcourse or terminal guidance. The mission repeats, allowing the microsat to image the target satellite from various angles and under multiple lighting conditions until the mission is completed or the fuel supply is exhausted.

In the most general repeated-intercept mission, the optimal strategy is that which provides the greatest number of flyby opportunities of a single target satellite within constrained budgets of time and fuel. Other considerations for this optimal strategy will include quality of image, certainty of imaging opportunity, and risk of collision. Quality of image includes lighting conditions and distance to and orientation of the target. Image opportunity certainty is related to the confidence that the target will be within the imaging field of view at a distance to permit imaging. Collision risk involves the uncertainty of the relative position and velocity of both the microsat and target. For some applications, the target point is some specified standoff distance from the target satellite. For this paper, a simplified cost function is used that presumes the targeted point is the location of the target satellite itself. Extension to the more general problem of targeting a standoff point is straightforward. Conditions of optimality are limited to the miss distance and fuel required for the first repeat intercept. So to summarize, this paper determines the instantaneous maneuver to perform at the condition of an initial intercept to affect a repeat intercept approximately one revolution later with a minimized combination of fuel and miss distance.

Assumptions and Definitions

Throughout this work, a nonmaneuvering target is assumed. The maneuvers performed by the microsat are of such duration as to be assumed impulsive; that is, they occur over an infinitesimal duration. As such, the maneuver results in an instantaneous change to the velocity components of the satellite state vector without changing the position; hence, the term delta-v. The satellites propagate according to a geopotential model accurate to J_6 and with atmospheric drag with no stochastic effects. This work is intended in part to develop an understanding of the solution space for the repeated-intercept problem. As such, uncertainties such as imperfect state or dynamics knowledge are not included. The microsat is assumed to have perfect knowledge of its own orbit as well as that of the target. Finally, all maneuvers are also assumed to occur perfectly (no burn errors). Although not realistic models, these assumptions simplify the problem and allow a better focus on the dynamics of the repeated-intercept geometry, providing a buildup approach for future work. In this paper, the microsat is assumed to have initially intercepted the target; that is, the target and microsat are collocated at the initial time.

Some definitions are appropriate here for clarification. Satellite rendezvous requires a vehicle to closely match its position and velocity to that of a target satellite. In these cases, the positions must both lie within some conjunction distance. The size of this distance will vary, depending on the mission, but will typically be on the order of 50–500 m. An intercept requires only the positions to be momentarily close. In general, the velocity vectors are not close during an intercept. The terms maneuver, burn, and delta-v will all be used to describe an application of force (thrust) delivered by a rocket motor to the satellite for a short time period.

Cost Function and Control Variables

The quality of a navigation solution for an intercept can be described by the miss distance and fuel required. Each of these can be represented by the three components of a Cartesian vector. These six components are then grouped to form a scalar cost function L, which is a function of the control variables $\boldsymbol{u}$. We then seek to minimize this scalar cost function over the control space. The scalar cost function is a weighted combination of the miss distance and fuel required and is formed as a sum of the squares of their normalized Cartesian elements.

$$L(\boldsymbol{u}) = \sum_{i=1}^{n} \left\{ \left(\frac{f_i(\boldsymbol{u})}{s_i} \right)^2 \right\} \tag{1}$$

Note that although each f_i could be defined to include the s_i terms and the exponent, the form of Eq. (1) allows a more natural definition of each f_i and also facilitates more flexible code.

Again, for this paper, the parameters of interest are the miss distance at the intercept time, measured in meters (m), and the amount of fuel required for the delta-v, measured in meters per second (m/s). These parameters are functions of the initial orbits and the control vector $\boldsymbol{u}$. With the cost function in the form of Eq. (1), we can define f_1, f_2, and f_3 as the Cartesian components of the vector miss distance at the intercept time using the following.

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \boldsymbol{R}_{\mathrm{ms}}(t_{\mathrm{int}}) \boldsymbol{R}_{\mathrm{tgt}}(t_{\mathrm{int}}) \tag{2}$$

where $\boldsymbol{R}_{\mathrm{ms}}(t_{\mathrm{int}})$ is the position vector of the microsat at the intercept time and $\boldsymbol{R}_{\mathrm{tgt}}(t_{\mathrm{int}})$ is that of the target. Similarly, the portions of the cost function associated with the delta-v are also expressed as their Cartesian components using

$$\begin{bmatrix} f_4 \\ f_5 \\ f_6 \end{bmatrix} = \Delta \boldsymbol{V} \tag{3}$$

In addition to these f_i, we specify the following scaling parameters.

$$s_1 = s_2 = s_3 = 100 \text{ m} \qquad s_4 = s_5 = s_6 = 100 \text{ m/s} \tag{4}$$

This completes the definition of the cost function.

The four control variables are assembled into a single control vector $\boldsymbol{u}$. Besides being part of the cost function, the Cartesian components of the delta-v applied at the initial time also represent three of the control variables. The time of the intercept is the fourth control variable and we explicitly present the complete control vector here.

$$\boldsymbol{u} = [\, \Delta V_x \quad \Delta V_y \quad \Delta V_z \quad t_{\mathrm{int}} \,]^T \tag{5}$$

Introduction to T-Matrix Navigation

T-matrix navigation (TMN) is a local gradient-based search algorithm developed to efficiently solve the repeated-intercept problem described previously. This section describes the difficulties associated with the application of classic forms of satellite navigation to the repeated-intercept problem, develops TMN in the form required for this problem, and relates TMN to the classic optimization methods of least squares and steepest descent.

Difficulty with Classic Methods

The classic approaches to finding a navigation solution for rendezvous or intercept generally begin with a solution to Lambert's problem. Lambert's problem is fundamentally the means to finding an orbit that passes through two known positions given some fixed time of travel. When combined with knowledge of both the target and interceptor position as functions of time, algorithms can be developed that determine the best maneuver for the interceptor to meet mission requirements. This generally involves some tradeoff between the delta-v required and the transfer time. Unfortunately, solution methods to Lambert's problem are inadequate for the repeated-intercept problem for two reasons. First, solutions to Lambert's problem encounter singularities when the position vectors at the initial and final time are collinear. When this occurs, there is not a unique orbit connecting the boundary positions even with a fixed transfer time. Second, and more important, solutions to Lambert's problem almost without exception assume only Keplerian dynamics. Fortunately, the difficulties posed by the repeated-intercept problem also present an opportunity. It is clear that if two-body gravitation was the only force, the solution would be for the interceptor to simply match the orbital period of the target. Therefore, it is likely that the desired maneuver is a small perturbation of this period-matching maneuver. This insight led to the development of TMN, which can be considered an extension of the orbit-determination methodology of least-squares differential correction (LSDC) as applied to the navigation problem.

Navigation Using Least-Squares Differential Correction

Classic orbit determination using LSDC involves determining a reference state for a satellite at an epoch time that, when propagated to times of actual observations, will best fit those observations in a least-squares sense. The dynamics are typically linearized, as are the observation relations, requiring an iterative process of making differential corrections to the reference state to cause the predicted observations to better match the actual observations. The method of navigation using least squares treats the desired intercept position as an observation and treats the desired intercept time as the fixed time of that observation. Using this approach, we solve for the initial state vector at time zero (the reference state), which will match the desired position at the intercept time. This initial state vector will be the required postmaneuver state. Only the velocity components of the reference state can be altered to fit the observation; the position components are the initial position vector and are fixed. Because there are three Cartesian components of target position at the intercept time, we must minimize the final error between microsat and target position in a least-squares sense. The maneuver required is then the difference between the velocity of the microsat reference state (postmaneuver) and the velocity before the maneuver. If this maneuver is performed, the microsat will follow a trajectory that will intercept the target at the desired intercept time (one revolution later).

The method of least squares can accommodate any perturbation desired. It is applicable in the very common situation in which the cost function is a sum of squares. Here, the cost function is the sum of the squares of the Cartesian elements of the miss distance and of the delta-v. In the classic least-squares implementation, the cost function is composed only of the residual errors at the observation times. However, in TMN, the cost of the maneuver applied at the initial time is included as part of the cost function.

Although the method of least squares is well-described in the literature, it is appropriate here to further explain this application by stepping through the present problem and describing the relevant equations. We begin with the state vectors of the target and microsat at time zero, immediately before the burn.

$$\boldsymbol{X}_{\text{ms}}(t_0^-) = \begin{bmatrix} R_{\text{ms},x}(t_0^-) \\ R_{\text{ms},y}(t_0^-) \\ R_{\text{ms},z}(t_0^-) \\ V_{\text{ms},x}(t_0^-) \\ V_{\text{ms},y}(t_0^-) \\ V_{\text{ms},z}(t_0^-) \end{bmatrix} \qquad \boldsymbol{X}_{\text{tgt}}(t_0) = \begin{bmatrix} R_{\text{tgt},x}(t_0) \\ R_{\text{tgt},y}(t_0) \\ R_{\text{tgt},z}(t_0) \\ V_{\text{tgt},x}(t_0) \\ V_{\text{tgt},y}(t_0) \\ V_{\text{tgt},z}(t_0) \end{bmatrix} \tag{6}$$

The − superscript indicates that this is the microsat state before the burn. After the burn, the microsat state will be

$$\boldsymbol{X}_{\text{ms}}(t_0^+) = \begin{bmatrix} R_{\text{ms},x}(t_0^-) \\ R_{\text{ms},y}(t_0^-) \\ R_{\text{ms},z}(t_0^-) \\ V_{\text{ms},x}(t_0^-) + \Delta V_x \\ V_{\text{ms},y}(t_0^-) + \Delta V_y \\ V_{\text{ms},z}(t_0^-) + \Delta V_z \end{bmatrix} \tag{7}$$

Both vehicles will follow predictable trajectories according to the equations of motion (EOM), so we can numerically integrate each trajectory to the intercept time. At this point, the three-dimensional vector miss distance is determined from

$$\boldsymbol{r}_f \equiv \boldsymbol{R}_{\text{ms}}(t_f) - \boldsymbol{R}_{\text{tgt}}(t_f) \tag{8}$$

To implement the least-squares algorithm, the sensitivity of $\boldsymbol{r}_f$ to changes in the microsat initial velocity is determined using the state transition matrix (STM). The STM is a 6×6 sensitivity matrix describing the effects on the end state caused by infinitesimal variations in the initial state.

$$\Phi(t_f, t_0) = \left[\frac{\partial X_{\text{ms}}(t_f)}{\partial X_{\text{ms}}(t_0)}\right] \tag{9}$$

The STM can be obtained by making small variations to the initial conditions and then propagating to determine their effects. A more elegant approach involves numerically integrating the equations of variation. This is usually conducted simultaneously with the integration of the preceding EOM. Whichever method is used, we extract the upper-right 3×3 submatrix Φ_{sub}^{u-r}, which describes the first-order (linearized) effects on the final position due to changes in the initial velocity. In some texts, the symbol T is used to represent this submatrix, and we will refer to it as the T matrix for the remainder of this paper.

$$T = \Phi_{\text{sub}}^{u-r}(t_f, t_0) = \frac{\partial \boldsymbol{r}_f}{\partial \Delta \boldsymbol{V}} \tag{10}$$

The predicted vector miss distance and the T matrix are combined to determine a differential correction to the initial velocity vector using

$$\delta \Delta \boldsymbol{V} = (T^T Q^{-1} T)^{-1} T^T Q^{-1} \delta \boldsymbol{r}_f \tag{11}$$

where Q serves as a weighting matrix based on the accuracy of the observations or the desired tolerance of the navigation solution. Equation (11) is used to determine the required adjustment to the nominal delta-v. Because the adjustment relies on the linearized Eq. (10), it must be iterated. That is, the adjustment is applied to the microsat initial velocity vector, the satellite states are again propagated to the intercept time, a new $\boldsymbol{r}_f$ and a new STM are determined, and another adjustment is calculated. This repeats until the adjustments converge to a sufficiently small value.

Classic least-squares orbit-determination techniques assume that the observation times are known, which effectively eliminates time from the reference state. It also assumes that the elements of the residual vector are functions of, but not the same as, the elements of the reference state. Given these limitations, Eq. (10) provides the sensitivity matrix of the elements that we care about, $\boldsymbol{r}_f$, with respect to the elements of the reference state that we can control, $\Delta \boldsymbol{V}$.

Setting the intercept time to one orbit period after the initial time, least-squares navigation efficiently arrives at a navigation solution to the repeated-intercept problem by making differential corrections to the microsat initial velocity until a state is found that propagates, according to the EOM, to a position that matches the target at the intercept time. Unfortunately, because least squares does not penalize the magnitude of the initial maneuver, the algorithm very quickly, and very accurately, converges on the *rendezvous* solution. That is, the microsat performs a very large maneuver to match the target velocity at time zero, after which the two vehicles' positions are forever matched. Although this is a possible maneuver given sufficient fuel, it is clearly not the minimum fuel solution that we desire.

T-Matrix Navigation

This is the point at which T-matrix navigation parts from the preceding description of least squares. Instead of determining a reference state, TMN calls this a vector of control variables. In the preceding description, the control variable vector was the maneuver $\Delta \boldsymbol{V}$. The description of the change in final position assumed that the intercept time was fixed. By changing the initial velocity, we not only change the microsat position at the intercept time, but we also slightly change the time of closest approach. We can treat this time of closest approach (the intercept time) as an additional control variable and more efficiently iterate toward a solution. This is the equivalent of fitting a least-squares trajectory to a set of observations while allowing the recorded times of the observations to change. TMN also differs from classic least squares in that the same variables used in the control vector can also appear (explicitly) in the cost function. We wish to find the maneuver that minimizes the miss distance while also keeping its own magnitude to a minimum. We combine the elements of the cost function into an error vector $\boldsymbol{e}$ containing parameters that we would like to drive to zero. So we have

$$\boldsymbol{e} = [\, r_{f,x} \quad r_{f,y} \quad r_{f,z} \quad \Delta V_x \quad \Delta V_y \quad \Delta V_z \,]^T \tag{12}$$

We now develop the complete T matrix for this scenario. Equation (10) served as the complete T matrix for least squares. In TMN, the 6×4 T matrix is defined as

$$T \equiv \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{u}} = \begin{bmatrix} \frac{\partial \boldsymbol{r}_f}{\partial \Delta \boldsymbol{V}} & \frac{\partial \boldsymbol{r}_f}{\partial t_f} \\ \frac{\partial \Delta \boldsymbol{V}}{\partial \Delta \boldsymbol{V}} & \frac{\partial \Delta \boldsymbol{V}}{\partial t_f} \end{bmatrix} \tag{13}$$

In addition to Eq. (10), we also need to know the sensitivity of the final position error to changes in the intercept time. This is simply the 3×1 difference in final velocities.

$$\frac{\partial \boldsymbol{r}_f}{\partial t_f} = \boldsymbol{V}_{\mathrm{ms}}(t_f) - \boldsymbol{V}_{\mathrm{tgt}}(t_f) \tag{14}$$

The maneuver is not an explicit function of the intercept time, and so the lower right 3×1 portion of the T matrix is a column of zeros. The lower left is a 3×3 identity matrix. So we have

$$T = \begin{bmatrix} \Phi^{u-r}_{\mathrm{sub}}(t_f, t_0) & \boldsymbol{V}_{\mathrm{ms}}(t_f) - \boldsymbol{V}_{\mathrm{tgt}}(t_f) \\ \mathrm{I}_{3\times 3} & 0_{3\times 1} \end{bmatrix} \tag{15}$$

LSDC also includes a Q matrix that consists of the inverse of the squared scaling parameters from the cost function.

$$Q = \mathrm{diag}(s_1^{-2}, s_2^{-2}, \ldots, s_6^{-2}) \tag{16}$$

At this point, the algorithm proceeds by making the following small change to the control vector.

$$\delta \boldsymbol{u} = (T^T Q^{-1} T)^{-1} T^T Q^{-1} \boldsymbol{e} \tag{17}$$

This process repeats until convergence, which is achieved when the corrections to each element of the control vector approach zero, to within a desired tolerance. For this paper, the tolerance is 10^{-4} m/s and 10^{-3} s. At that point, we have located a local extremum of the cost function by finding the point at which

$$\frac{\partial L(\boldsymbol{u})}{\partial \boldsymbol{u}} \approx 0_{1\times 4} \tag{18}$$

Because our cost function is defined as a quadratic with nonnegative scaling coefficients, this extremum is a local minimum.

Initial Control Variable Estimate

An important requirement of T matrix navigation, as in least squares, is the need for an initial estimate of the control variables. This estimate must be close to the optimum solution to allow for convergence. In the repeated-intercept problem, an initial estimate is readily available. The maneuver required for intercept will be very close to the maneuver required to simply match the vehicles' periods. In subsequent maneuvers, after the second intercept, the initial delta-v estimate can be zero. Also, the intercept will occur at a time near one orbit after the first intercept. So the target's orbit period is used as the starting guess at the intercept time. It can be shown that both of these initial estimates are generally well within the region of convergence.

Comparison of TMN to the Method of Steepest Descent

Least squares can be categorized as a method of steepest descent, as described in [12–14]. In this method, out of a class called "numerical solution by a first-order gradient method" [13], the cost associated with an initial guess at the control vector is evaluated, along with the gradient of the cost with respect to the control. This gradient is typically evaluated numerically and locally at the value of the control. The control is then changed in the direction of the local gradient (hence the term *steepest descent*), which will then change the cost. Under certain conditions, such as continuity of the cost function (see [13] for more details), the new cost will be lower. This process is then iterated until a local minimum is found.

The size of the step taken at each iteration is one difference between various steepest-descent methods. The step size can be adjusted by adding a factor k to Eq. (17), giving

$$\delta \boldsymbol{u} = k(T^T Q^{-1} T)^{-1} T^T Q^{-1} \boldsymbol{e} \tag{19}$$

In the classic least-squares formulation, this constant is left out, leading to a step size of unity. A step of size unity in the direction of the local gradient would reach the optimum location if the cost function were purely quadratic across the region. In fact, for a system of linear equations, this is the case and this is precisely how we solve a set of simultaneous linear equations in linear algebra. The preceding formulation assumes that the curve of the solution space can be approximated by a (specific) local quadratic. In this application, that is (locally) a very accurate approximation and hence TMN converges quite rapidly given a reasonably accurate initial estimate.

In cases in which the solution space is very nonlinear, a nonunity value of k may be required. This prevents overshooting the optimum. Another technique involves a search along the direction of the local gradient by evaluating the cost function that would be achieved using different values of k until a minimum is found and then moving to that location. This technique is not as practical in this application, because each evaluation of the cost function requires an expensive numerical integration of the EOM.

Optimization Using *T*-Matrix Navigation

To demonstrate the utility of the iterative TMN algorithm, we consider three cases. These cases represent potential navigation problems for a microsat to perform the repeated-intercept mission. Again, in all cases, we assume a nonmaneuvering target and that the target and microsat are initially collocated.

Initial Orbits

For Case 1, the two vehicles' initial orbital elements are shown in Table 1.

The target elements were chosen more or less randomly to produce an elliptical inclined LEO. The microsat elements were formed by modifying the target's initial state vector. The microsat initial position matches that of the target, whereas the microsat initial velocity vector is rotated relative to that of the target by 0.1 deg downward in the orbit plane and 35 deg out of the orbit plane. The microsat velocity vector is also 38.6 m/s larger than that of the target (7.755 km/s vs 7.716 km/s), which sets the initial perigee at approximately 350 km. Starting with the target state vector and modifying the velocity components is simply a convenient way to generate another elliptical inclined LEO with a matched starting position. The orbits for the second case are shown in Table 2.

This initial geometry is simply the resulting intercept after application of the optimal solution to case 1 (not shown yet). This

Table 1 Initial orbital elements, case 1

Orbital element	Target	Microsat
Semimajor axis, Earth radii	1.0600	1.0709
Eccentricity	0.0050	0.0152
Inclination, deg	15.0000	40.0519
Long. of ascending node, deg	45.0000	106.3808
Argument of perigee, deg	70.0000	13.3168
True anomaly at epoch, deg	10.0000	10.0182

Table 2 Initial orbital elements, case 2

Orbital element	Target	Microsat
Semimajor axis, Earth radii	1.0600	1.0594
Eccentricity	0.0050	0.0043
Inclination, deg	15.0000	40.0518
Long. of ascending node, deg	44.4955	105.9814
Argument of perigee, deg	70.7081	13.9787
True anomaly at epoch, deg	9.4002	9.3645

Table 3 Initial orbital elements, case 3

Orbital element	Target	Microsat
Semimajor axis, Earth radii	1.0500	1.0607
Eccentricity	0.0050	0.0248
Inclination, deg	15.0000	47.3499
Long. of ascending node, deg	335.0000	5.0842
Argument of perigee, deg	5.0000	316.8095
True anomaly at epoch, deg	35.0000	56.2635

case represents the problem of determining the navigation solution for the second repeat. The orbits for the third case are shown in Table 3.

The orbits of case 3 are distinct from the previous cases. This case was selected out of the many that were examined in this research because it reveals a difficulty with the repeated-intercept mission when restricted to maneuvers at the initial intercept time. This difficulty is not apparent in the first two cases. We will see that the final cost is high in case 3 because the microsat must maneuver in a direction of space not easily reached from its initial position. This is closely related to the singularity encountered in the case of attempting to raise perigee of a two-body orbit by maneuvering at the point of perigee.

Case 1 Results

Case 1 converges using T-matrix navigation in four iterations. The navigation solution is the final control vector.

$$\boldsymbol{u} = \begin{bmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \\ t_f \end{bmatrix} = \begin{bmatrix} 31.1080 \text{ m/s} \\ 12.1224 \text{ m/s} \\ -27.9798 \text{ m/s} \\ 5513.2737 \text{ s} \end{bmatrix} \tag{20}$$

This is a delta-v magnitude of 43.56 m/s. The final miss distance is 1.93 m and the total scaled (unitless) cost $L(\boldsymbol{u})$ is 0.190.

Case 2 Results

Case 2 converges using T-matrix navigation in three iterations. The navigation solution is

$$\boldsymbol{u} = \begin{bmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \\ t_f \end{bmatrix} = \begin{bmatrix} 0.5252 \text{ m/s} \\ -0.7729 \text{ m/s} \\ -0.2442 \text{ m/s} \\ 5513.2334 \text{ s} \end{bmatrix} \tag{21}$$

This is a delta-v magnitude of 0.97 m/s. The final miss distance is less than 1 m and the cost is 9.5×10^{-5}. These results indicate that with matched orbit periods only very small maneuvers are required to affect repeat intercepts.

Case 3 Results

Case 3 was chosen to illustrate the impact of restricting the initial maneuver time. TMN requires nine iterations to converge to the following navigation solution.

$$\boldsymbol{u} = \begin{bmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \\ t_f \end{bmatrix} = \begin{bmatrix} -88.8072 \text{ m/s} \\ -53.2286 \text{ m/s} \\ -44.7806 \text{ m/s} \\ 5434.9662 \text{ s} \end{bmatrix} \tag{22}$$

This is a delta-v magnitude of 112.81 m/s and results in a miss distance of 23.95 m. This corresponds to a fairly high scaled cost of 1.330.

Discussion of Results

The results from case 1 indicate that the optimal three-dimensional maneuver of Eq. (20) is only 0.016 deg out of the microsat orbit plane. This points to the use of a coplanar solution search as an efficient suboptimal navigation strategy for use as an initial guess, because the additional degree of freedom is not very useful to the minimization problem. However, including fuel in the cost function does force the algorithm to arrive at an efficient navigation solution to affect the repeat intercept without driving a large-magnitude rendezvous solution, as least squares would. Finally, note that adjusting the relative scales s_i would give a result that is either more accurate while using more fuel or less accurate while conserving fuel.

Case 2 revealed one very significant aspect of the repeated-intercept problem. The fuel required to affect a repeated intercept is minimal once the orbit periods have been matched. This indicates that once the first repeated intercept is accomplished, a microsat can perform many subsequent intercepts within a limited delta-v budget.

The results from case 3 were quite different. Although the orbits were fairly similar to those of cases 1 and 2, case 3 shows a situation in which the target location is not easily reached from the starting position. More precisely, the upper-right portion of the STM has a nontrivial null space [15]. At the final iteration, this matrix is

$$\Phi_{\text{sub}}^{u-r}(t_f, t_0) = \begin{bmatrix} -1.55 & 3.56 & 4.01 \\ 3.61 & -8.31 & -9.35 \\ 4.05 & -9.32 & -10.50 \end{bmatrix} \tag{23}$$

in canonical units, in which the distance unit is the mean equatorial radius of the Earth and the time unit is 13.447 min. This matrix gives the effect on the final position caused by the initial delta-v. By inspection, all three columns appear nearly collinear, indicating that all three delta-v components tend to produce position changes along the same direction. Producing a position change in a direction orthogonal to this is very difficult.

The eigenvalues of this matrix provide a way to quantify this situation. The eigenvalues are -20.3505, -0.0054, and -0.0040. This indicates that one direction of space is reached $20/0.004 = 5000$ times easier than another direction. This ratio of largest to smallest eigenvalues is a quick way to test whether the chosen time of maneuver allows an effective span of control. We can also look at the final position error vector, in meters,

$$\boldsymbol{r}_f = \begin{bmatrix} 22.979 \\ 5.576 \\ 3.806 \end{bmatrix} \tag{24}$$

and see that it indeed points in a direction of space not easily reached. Examining the dot product between a unit vector in the direction of Eq. (24) and unit vectors in each of the three directions spanned by Eq. (23), we find that the largest is 0.0008.

The conclusion to draw in this situation is that the microsat is attempting to perform a maneuver at a constrained initial time to affect a change in a direction not easily within its span of control, the eigenspace of Eq. (23) [16]. Although the submatrix is not singular, it is close enough that we see a large cost penalty manifested in the amount of fuel required to correct a final position error of less than 24 m. Even with the intercept time as a degree of freedom, we are not able to efficiently repeat the intercept of this target with a burn constrained to occur at the initial time.

It is important to note here that the nearly singular matrix in case 3 is not very different from the equivalent matrices of cases 1 and 2 and so could not be used to predict this difficulty in advance. In all cases when the maneuver is constrained to occur at the initial time, the eigenvalues will exhibit these very high ratios. In fact, the same analysis applied to case 1 yields a ratio of eigenvalues even higher than in case 3. Why, then, does case 3 result in such a high cost? The answer lies in the directions of space easily reached by the maneuver. In case 3, the initial velocity vector is rotated in the orbit plane relative to the target by a full degree, 10 times more than in case 1. This generates a more significant radial velocity component relative to the target. This radial velocity creates the problem. It results in a slight change in altitude of the microsat at the intercept time. This altitude displacement must be corrected by the maneuver. It is effectively a radial displacement, which is precisely the direction of space that is difficult to reach. So in cases 1 and 2, the microsat is limited in its reach but the target happens to lie within that reach. In case 3, because of the larger component of relative radial velocity, the target lies in a direction not easily within the microsat's reach. The final miss distance of roughly 24 m lies along a vector within 1 deg of collinear with the position vector.

To emphasize, this situation is a product of the physics underlying the mission, not a limitation of TMN. In fact, this singularity is precisely the reason that solutions to Lambert's problem fail when applied to the repeated-intercept mission. TMN allows solutions in some cases, but cannot completely avoid the singularity. To do this, we must introduce the time of burn as an additional degree of freedom to the navigation strategy.

Conclusions

T-matrix navigation is an efficient method of determining an initial maneuver to affect a repeated intercept when the maneuver is constrained to occur at the time of an initial intercept. In some cases, constraining the maneuver to occur at a specific time will force larger fuel requirements than solutions with unconstrained burn times. This paper only showed that TMN was appropriate for the initial maneuver at the constrained burn time. Future research will consider burns of unconstrained times as well as midcourse and terminal guidance. When combined, these guidance algorithms allow an efficient navigation strategy that will permit a microsatellite to image a target satellite under various angles and under multiple lighting conditions during repeated intercepts, thus providing an on-orbit inspection capability without matching orbital planes.

References

[1] Alfano, S., and Fosha, C. E., Jr., "Hypervelocity Orbital Intercept Guidance Using Certainty Control," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 3, May–June 1991, pp. 574–580.

[2] Alfano, S., "Two Variations of Certainty Control," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 4, 1992, pp. 1040–1043.

[3] Bate, R. R., Mueller, D. D., and White, J. E., *Fundamentals of Astrodynamics*, Dover, New York, 1971.

[4] Lawton, J. A., and Byrum, C. A., "Antitactical Ballistic Missile Global Effectiveness Model (AGEM) Intercept Algorithm," Strategic and Space Systems Department, U.S. Naval Surface Warfare Center, Rept. NSWDD/TR-92/527, Dahlgren, VA, July 1994.

[5] Lu, P., "Intercept of Nonmoving Targets at Arbitrary Time-Varying Velocity," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, Jan.–Feb. 1998, pp. 176–178.

[6] Newman, B., "Spacecraft Intercept Guidance Using Zero Effort Miss Steering," AIAA Paper 93-3890-CP, 1993.

[7] Ng, L. C., Breitfeller, E., and Ledebuhr, A. G., "An Optimal t-Delta-V Guidance Law for Intercepting a Boosting Target," Lawrence Livermore National Lab., Rept. UCRL-JC-148995, Livermore, CA, 2002.

[8] Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, 2nd ed., Microcosm, El Segundo, CA, 2001.

[9] Youmans, E. A., and Lutze, F. H., "Neural Network Control of Space Vehicle Intercept and Rendezvous Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, Jan.–Feb. 1998, pp. 116-121.

[10] Zes, D., "Exo-Atmospheric Intercept Using Modified Proportional Guidance with Gravity Correction for Coast Phase," AIAA Paper 94-0209, Jan. 1994.

[11] Zes, D., "Exo-Atmospheric Intercept with J2 Correction," AIAA Paper 1998-4305, 1998.

[12] Bryson, A. E., Jr., and Ho, Y.-C., *Applied Optimal Control, Optimization, Estimation, and Control*, Hemisphere, Levittown, PA, 1975.

[13] Bryson, A. E., Jr., *Dynamic Optimization*, Addison Wesley, Menlo Park, CA, 1999.

[14] Gerald, C. F., and Wheatley, P O., *Applied Numerical Analysis*, 7th ed., Addison Wesley, Boston, 2004.

[15] Hoffman, K., and Kunze, R., *Linear Algebra*, 2nd ed., Prentice–Hall, Upper Saddle River, NJ, 1971.

[16] Kreyszig, E., *Advanced Engineering Mathematics*, 8th ed., Wiley, New York, 1999.

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